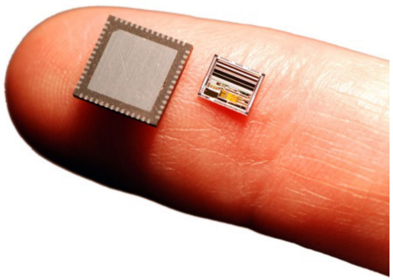
intan TECHNOLOGIES, LLC

RHD2000



FAQ: Amplifier Filter Characteristics

25 June 2013

Q:

What are the gain and phase characteristics of the RHD2000 amplifiers as a function of signal frequency?

A:

The amplifiers at the core of RHD2000 digital electrophysiology interface chips have a band-pass behavior, passing signals between a user-programmable lower cutoff frequency (f_L) and upper cutoff frequency (f_H). As with all physically realizable circuits, the amplifiers do not have a "brick wall" filtering characteristic; the gain of the amplifiers decreases smoothly beyond the user-specified bandwidth, asymptotically approaching zero. The cutoff frequencies f_L and f_H merely denote points where the gain has decreased by a factor of 3 dB ($1/\sqrt{2}$). Additionally, signals near the cutoff frequencies are subject to phase shifts due to the action of the filters.

Upper Cutoff Frequency (f_H)

The upper limit of the amplifier pass band has a three-pole 3rd-order Butterworth low-pass filter characteristic. This filter is described by the following complex transfer function:

$$H_H(s) = \frac{1}{\left(\frac{s}{\omega_H}+1\right)\left(\left(\frac{s}{\omega_H}\right)^2+\frac{s}{\omega_H}+1\right)}$$

where $s = j\omega$, $j^2 = -1$, and ω is frequency in radians/second. The upper cutoff frequency is expressed as $\omega_H = 2\pi f_H$. Solving for amplitude (i.e., gain), and writing frequency in units of Hertz for convenience, we get:

$$|H_H(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_H}\right)^6}}$$

The gain is close to unity for frequencies much less than f_H, and then decreases rapidly (as $1/f^3$, or 60 dB/decade) for frequencies above f_H. At the cutoff frequency f_H, the gain is $1/\sqrt{2}$ = -3 dB. (In the filter analysis presented here, we will describe the amplifier transfer function normalized to a gain of one in its pass band. This corresponds to an electrode-referred signal level of 0.195 µV/LSB at the on-chip analog-to-digital converter.)

Note: To convert gain expressed linearly (e.g., in V/V) to and from dB (decibels) use the following relationships:

$$\mathrm{Gain_{dB}} = 20 \cdot \log_{10} \mathrm{Gain_{V/V}}$$

$$\mathrm{Gain_{V/V}} = 10^{\mathrm{Gain_{dB}}/20}$$

Solving the 3rd-order Butterworth filter characteristic for phase angle, we get:

$$\angle H_H(f) = -\tan^{-1}\left(\frac{2\left(\frac{f}{f_H}\right) - \left(\frac{f}{f_H}\right)^3}{1 - 2\left(\frac{f}{f_H}\right)^2}\right)$$

This function approaches 0° for frequencies much less than f_H and asymptotically approaches -270° for signals much greater than f_H. At the cutoff frequency f_H, the phase is -135°, meaning signals at this frequency experience a phase lag of 135°.

Lower Cutoff Frequency (f_L)

The lower limit of the amplifier pass band has a simple one-pole high-pass filter characteristic. This filter is described by the following complex transfer function:

$$H_L(s) = \frac{s}{s + \omega_L}$$

where $\omega_L = 2\pi f_L$ is the lower cutoff frequency.

Solving for amplitude (i.e., gain), and writing frequency in units of Hertz for convenience, we get:

$$|H_L(f)| = \frac{1}{\sqrt{1 + \left(\frac{f_L}{f}\right)^2}}$$

The gain is close to unity for frequencies much less higher than f_L, and then decreases proportionally (20 dB/decade) for frequencies below f_L. At the cutoff frequency f_L, the gain is $1/\sqrt{2}$ = -3 dB.

Solving for phase angle, we get:

$$\angle H_L(f) = \tan^{-1}\left(\frac{f_L}{f}\right)$$

This function approaches +90° for frequencies much less than f_L and asymptotically approaches 0° for signals much greater than f_L. At the cutoff frequency f_L, the phase is +45°, meaning signals at this frequency experience a phase lead of 45°.

DSP Offset Removal Filter (f_{DSP})

The RHD2000 chips contain an optional DSP (digital signal processing) high-pass filter to remove small dc voltage offsets that accumulate in the analog amplifier circuitry. (The amplifiers are coupled to electrodes through on-chip series capacitors that block all of the large electrode-tissue potentials that can be hundreds of millivolts in amplitude, but several millivolts of additional offset can creep into the signal as it passes through the amplifier circuitry.)

This digital filter closely approximates the characteristics of the continuous-time one-pole high-pass filter described above for the lower cutoff frequency f_L. The gain and phase equations for the DSP offset removal filter may be taken from the previous section, substituting f_{DSP} for f_L.

Complete Amplifier Transfer Function

The complete amplifier transfer function is calculated by multiplying the three complex functions described above. This is equivalent to multiplying their amplitudes (gains) and summing their phase angles:

$$\left|H_{\text{amp}}(f)\right| = |H_L(f)| \cdot |H_H(f)| \cdot |H_{\text{DSP}}(f)| = \frac{1}{\sqrt{1 + \left(\frac{f_L}{f}\right)^2}} \cdot \frac{1}{\sqrt{1 + \left(\frac{f}{f_H}\right)^6}} \cdot \frac{1}{\sqrt{1 + \left(\frac{f_{\text{DSP}}}{f}\right)^2}}$$

$$\angle H_{\text{amp}}(f) = \angle H_L(f) + \angle H_H(f) + \angle H_{\text{DSP}}(f) = \tan^{-1}\left(\frac{f_L}{f}\right) - \tan^{-1}\left(\frac{2\left(\frac{f}{f_H}\right) - \left(\frac{f}{f_H}\right)^3}{1 - 2\left(\frac{f}{f_H}\right)^2}\right) + \tan^{-1}\left(\frac{f_{\text{DSP}}}{f}\right)$$

MATLAB Calculation of Gain and Phase

The following MATLAB m-file (which should be saved as **ideal_transfer_function.m**) can be used to compute gain and phase characteristics for a vector containing frequency values. The cutoff frequencies f_L, f_H, and f_{DSP} are passed to the function. If the DSP offset removal filter is disabled, f_{DSP} should be set to zero.

```
function [gain, phase] = ideal_transfer_function(f, fL, fH, fDSP)

% [gain, phase] = ideal_transfer_function(f, fL, fH, fDSP)
%
% f is a vector containing frequencies; fL and fH are lower and upper
% cutoff frequencies (in Hz); fDSP is the DSP offset removal filter
% frequency.
%
% Example usage to plot gain and phase from 0.1 Hz to 100 kHz when
% fL = fDSP = 1 Hz and fH = 7.5 kHz:
%
% >> f = logspace(log10(0.1), log10(100e3), 1000);
% >> [gain, phase] = ideal_transfer_function(f, 1, 7.5e3, 1);
% >> figure(1);
% >> semilogx(f, gain);
% >> figure(2);
% >> semilogx(f, phase);

L = length(f);

for i=1:L
    gain(i) = 1 / (sqrt(1 + (f(i)/fH)^6) * sqrt (1 + (fL/f(i))^2));
    phase(i) = atan(fL/f(i)) - atan((2*(f(i)/fH) - (f(i)/fH)^3)/(1 - 2*(f(i)/fH)^2));
end

if (fDSP > 0)
    for i = 1:L
        gain(i) = gain(i) / sqrt(1 + (fDSP/f(i))^2);
        phase(i) = phase(i) + atan(fDSP/f(i));
    end
end

gain = 20 * log10(gain);    % convert gain to decibels
phase = (180/pi) * phase;   % convert phase from radians to degrees
phase = unwrap(phase);      % unwrap phase

end
```

Example Gain and Phase Plots

Figures on the following page provide plots of gain and phase vs. frequency for RHD2000 amplifiers set to a pass band of 1 Hz – 10 kHz. Each plot is shown both with the DSP offset removal filter disabled and enabled (with f_{DSP} = 1 Hz). Note that setting f_{DSP} = f_L will yield a normalized gain of -6 dB at that frequency due to the combined action of two high-pass filters. This pushes the effective -3 dB cutoff frequency up by a factor of approximately 1.6.

The MATLAB code provided above may be used to generate plots similar to the ones shown here. For example:

```
>> f = logspace(log10(0.1), log10(100e3), 10000);
>> [gain, phase] = ideal_transfer_function(f, 1, 7.5e3, 0);
>> figure(1); semilogx(f, gain);
```

```
>> figure(2); semilogx(f, 10.^(gain/20));
>> figure(3); semilogx(f, phase);
```

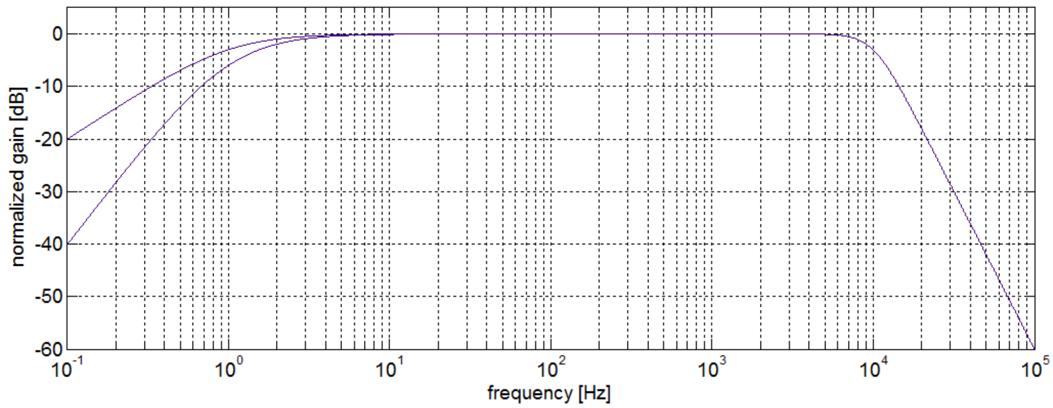


Figure 1. Normalized gain (plotted logarithmically in dB) vs. frequency plot for RHD2000 amplifier with f_L = 1 Hz and f_H = 10 kHz. Two plots show gain with the DSP offset removal filter disabled (top curve) or enabled (bottom curve) with f_{DSP} = 1 Hz. Setting the DSP filter to the same frequency as f_L pushes the effective -3 dB lower cutoff frequency up to 1.6 Hz in this case.

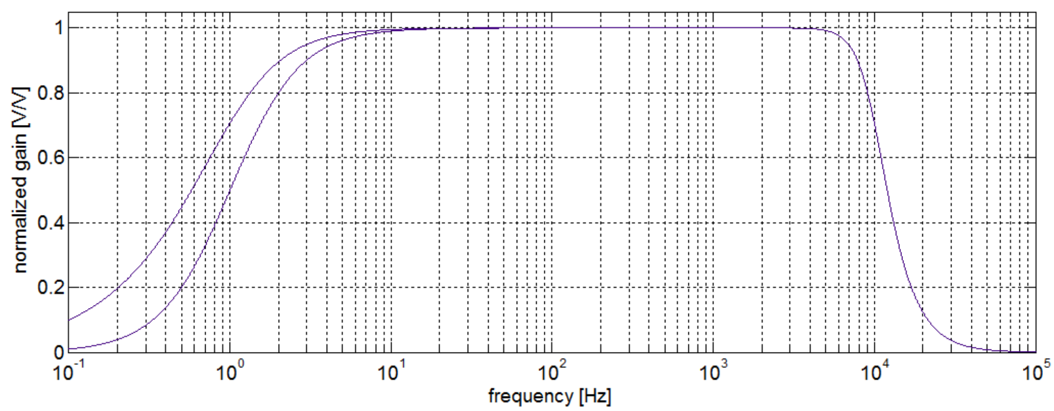


Figure 2. Normalized gain (plotted linearly) vs. frequency plot for RHD2000 amplifier with f_L = 1 Hz and f_H = 10 kHz. Two plots show gain with the DSP offset removal filter disabled (top curve) or enabled (bottom curve) with f_{DSP} = 1 Hz.

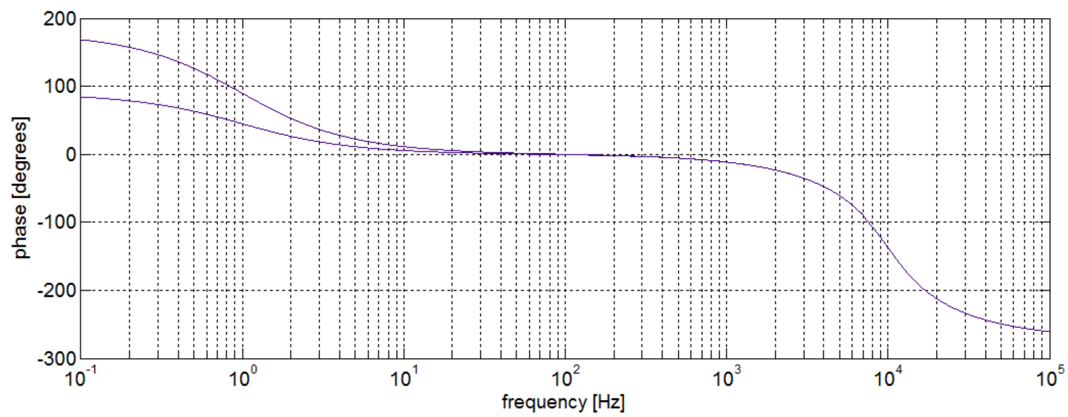


Figure 3. Phase vs. frequency plot for RHD2000 amplifier with f_L = 1 Hz and f_H = 10 kHz. Two plots show gain with DSP offset removal filter disabled (bottom curve) or enabled (top curve) with f_{DSP} = 1 Hz. The DSP offset removal filter adds an extra 90° of phase lead to extremely low frequency signals.